1. **Plato**

1.1. **Ontology**

Plato is an ontological realist: mathematical entities exist, and are mind-independent.

There is the world of *Being*, which consists of the *Forms*, and a world of *Becoming*, which consists of physical entities and appearances.

- We never encounter perfect justice, beauty, … circles in the world of Becoming, yet we can have clear ideas about these perfect entities are.
- This is because these perfect entities (Justice, Beauty, …)—what Plato called “the Forms”—are not in the world of Becoming. They are in a separate world of *Being*.
  - Mathematical objects are part of the world of *Being*, but unclear if they are Forms, or “reflections” of Forms.
- Forms are eternal and unchanging.

Argued for ontological realism using something like Benacerraf’s first horn (see previous handout).

1.1.1. *Another argument* (page 55)

In-class exercise

1.2. **Semantics**

Plato is a truth-value realist: some mathematical statements are true, and all mathematical truth-values are objective. (Truth-value antirealism is a relative latecomer, so this is taken for granted.)

1.3. **Epistemology**

Knowledge of the Forms can be gained *a priori* through:

- Careful reasoning and reflection
- “Recollection”: in our pasts, our souls had direct contact with the world of *Being*, and we can come to remember the Forms.

In both cases, Plato held that human minds are capable of interacting with both the world of *Being* and the world of *Becoming*.

1.3.1. *Argument*

P1. Geometrical statements are not about the physical world (from above).

P2. *Only those things that are about the physical world can be known through the senses.*

C1. ∴ Geometrical statements can only be known through something other than the senses.

P3. *Anything known through something other than the senses is *a priori*.*

C2. Geometrical statements are only known *a priori*.

1.4. **Philosophy of science**

Mathematical objects are approximated in the physical world.

However, there is still a question as to how the world of *Being* interacts with the world of *Becoming*.

- There are only analogies (physical reality is a “reflection” of Formal reality) and mysterious relations (particular physical objects “participate” in their corresponding Forms)

Later Plato (Timaeus): the physical world is created geometrically from the five Platonic solids.

2. **Aristotle**

2.1. **Ontology**

There are two interpretations of Aristotle:

- On one interpretation, he’s an ontological realist who holds that we access mathematical entities through a process of *abstraction*. (Shapiro calls this “abstractionism.”)
- On another interpretation, he’s an ontological antirealist who holds that we posit mathematical entities as useful *fictions*. (Shapiro calls this “fictionalism.”)

2.1.1. *Aristotle as realist/abstractionist*

Aristotle is widely regarded as an ontological realist, and also believes that numbers are Forms. However, he has a very different theory of the Forms.

- Unlike Plato, Aristotle did not believe in two worlds (Being and Becoming).
- Rather, for Aristotle, the Form of *F* exists *within* all of the objects that are *F*.
  - Ex. The Form of Beauty is what all beautiful things have in common.
2.1.2. Aristotle as antirealist/fictionalist

However, if Forms don’t exist over and above the entities that possess them, then in what sense do they exist at all? Skeptical answers to this question have led to an alternative interpretation of Aristotle, in which he is not committed to the existence of mathematical Forms, and hence not an ontological realist.

- On this antirealist interpretation, we do not abstract away non-mathematical properties of objects to get at the mathematical Forms.
- Instead, we ignore these non-mathematical properties, and mathematics allows us to develop ideas that can be applied to objects that can be ignored in the same way. (Ex. Brass and wooden spheres on p.68)
- This does not require any mathematical property/entity to exist, since these are simply physical properties. “…the postulation of geometric objects is harmless, since the real physical sphere also has all of those properties we attribute to the postulated sphere.” (p. 68)
- While we are free to speak as if there are mathematical objects, that’s simply shorthand for the long list of physical objects that we can ignore in the same way.

2.1.3. A Platonist challenge: the mismatch problem

2.1.3.1. Abstractionist’s mismatch problem

P1. If abstractionism is true, then the physical imperfections of all mathematical entities are abstracted away.

P2. If an entity’s physical imperfections are abstracted away, then it is eternal and unchanging.

P3. An entity that is eternal and unchanging is a Platonic entity.

C1. ∴ If abstractionism is true, then all mathematical entities are Platonic entities. (From P1-P3)

2.1.3.2. Fictionalist’s mismatch problem

P1. If fictionalism is true, then either there exist physical entities that are mathematically perfect, or it is possible to produce such entities using only physical tools.

P2. No physical entities are mathematically perfect.

P3. If it is only possible to produce (through physical tools) physical entities that are mathematically perfect, then mathematical objects are ideal objects that are eternal and unchanging.

P4. If mathematical objects are ideal objects that are eternal and unchanging, then they are Platonic entities.

C1. If fictionalism is true, then all mathematical entities are Platonic entities. (from P1-P4)

2.2. Semantics

Truth-value realism (truth-value antirealism is still a latecomer.)

2.3. Epistemology

Abstractionist epistemology: We gain mathematical knowledge by first abstracting away the non-mathematical properties from perceptible objects, and then reasoning on the basis of what remains after this process of abstraction.

Fictionalist epistemology: Mathematical knowledge is just like other knowledge of physical objects. However, since mathematical entities don’t exist, there is no knowledge of their existence.
2.3.1. Frege’s challenge to abstractionism

P1. If abstraction is the means by which we achieve arithmetic knowledge, then to count is to count objects that have no distinguishing properties whatsoever.

P2. For all objects $x$ and $y$, if $x$ and $y$ have no distinguishing properties, then $x$ and $y$ cannot be differentiated from each other.

P3. For all $x$ and $y$, if $x$ and $y$ cannot be differentiated from each other, then it is possible but unknowable that $x = y$.

P4. For all $x$ and $y$, if it is possible but unknowable that $x = y$, then it is unknowable whether $x$ and $y$ are one or two objects.

P5. If, for all $x$ and $y$, it is unknowable whether $x$ and $y$ are one or two objects, then counting is impossible.

P6. Counting is possible.

C1. $\therefore$ Abstraction is not the means by which we achieve arithmetic knowledge. (from P1-P6)

2.4. Philosophy of science/applicability

Mathematical properties are real properties of physical objects; consequently there’s no puzzlement as to how the physical world is mathematical.