1. **Kant's third crisis**

1.1. **The set-up**

Recall that Kant offers the following argument for the *a priori* of mathematical knowledge:

P1. We have sensory experiences that are objectively true/false with respect to space and time.

P2. If we have sensory experiences that are objectively true/false with respect to space and time, then we are committed to space and time having objective structures.

P3. *Definition*: Pure intuition = commitment to objective structure of space and time.

C1. ∴ We have pure intuitions. (from P1-P3)

P4. Knowledge about commitments is not acquired through our senses, i.e. it's *a priori*.

C2. Knowledge about pure intuitions is *a priori*. (from P3, P4)

P5. Mathematical knowledge is knowledge about pure intuitions.

C3. ∴ Mathematical knowledge is *a priori*. (from C2, P5)

So far so good, but Kant goes on:

P6. The objective structure of space is Euclidean.

As a result, he accepts the following:

C4. If we have sensory experiences that are objectively true/false with respect to space, then we are committed to space being Euclidean. (from P2, P6)

1.2. **The small crisis**

However, we have seen that C4 is false, for we are justified in thinking of the relevant sensory experiences as having an objective *non-Euclidean* structure. This leads to the following inconsistency in the previous argument:

P2. If we have sensory experiences that are objectively true/false with respect to space and time, then we are committed to space and time having objective structures.

P6. The objective structure of space is Euclidean.

Not-C4. We have sensory experiences that are objectively true/false with respect to space, but we are *not* committed to space being Euclidean.

It's pretty clear that the more central Kantian thesis is P2. So, one might think that Kant got the philosophy right, but the objective structure of space wrong (i.e. P6), and for understandable historical reasons.

1.3. **The bigger crisis**

However, this would be to overlook how we came to revise the geometry that currently underlies our best conception of the objective structure of space. We did not do so *a priori*. We did so because of the greater empirical success of relativity theory over classical mechanics. Hence, this also casts doubts on other parts of Kant's argument for the *a priori* of mathematical knowledge. We have a few options at this point:

- Deny that mathematical knowledge is *a priori*. (Mill, Quine (below), Devitt)
- Accept that mathematical knowledge is *a priori*, but deny that anything like Kant's argument is the reason why this is so. (Plato, Bonjour)
- Tweak our conceptions of the *a priori*, and the Kantian argument, so as to keep the general contours of the Kantian position intact. (Friedman, see below)

2. **Friedman's Kantianism**

For Kant, the *a priori* was [probably] *absolute* or [more speculatively] *relative* to the possibility of having sensory experiences that are objectively true/false with respect to space and time.

By contrast, for Friedman, the *a priori* is always relative to a theory. More precisely:

A proposition $p$ is *a priori* relative to a theory $T$ if it is not possible to empirically confirm/refute $p$, but $p$ is necessary to empirically test other parts $q$ of $T$. 
That force equals mass times acceleration is a priori relative to Newtonian mechanics. Let \( p \) be \( F=ma \). While this is taken as a “stipulation” in Newtonian mechanics (\( T \)), and hence not empirically testable, it does allow us to measure forces, which in turn allow us to test other parts of Newtonian mechanics, for instance the law of universal gravitation (\( q \)):
\[
F_g = \frac{Gm_1m_2}{r^2}
\]

2.1. **Back to the a priori argument**

On this view, our “pure intuitions” or “commitments” to the objective structure of space and time are *a priori relative to the prevailing theory*. Hence, what is *a priori* at one time may be *a posteriori* at another, if our prevailing theories shift.

Furthermore, *a priori* statements are stipulations: we simply take certain statements as brute definitions for the purposes of testing other statements.

2.2. **Scientific antirealism**

P1. If stipulations are allowed, then they do not contradict pre-existing facts.
P2. Among Friedman’s stipulations are Newton’s second law and the speed of light.
C1. \( \therefore \) There are no pre-existing facts about force’s relationship to mass and acceleration or the speed of light.

We could think otherwise about them and still have scientific theories that are as empirically successful as Newton and Einstein’s respectively.

2.3. **An Empiricist Objection to Friedman: Quine’s Master Argument**

P1. Let \( p \) be a proposition known *a priori*. (hypothesis for contradiction)
P2. Then we cannot falsify \( p \) by experience. (by 1, definition of *a priori*)
P3. \( p \), in conjunction other assumptions \( a_1, \ldots, a_n \) entails empirical hypotheses. (1st holist assumption)
P4. These empirical hypotheses could turn out to be false. (fact)
P5. If these empirical hypotheses turn out to be false, we’re free to reject \( p \) instead of any of the other assumptions \( a_1, \ldots, a_n \). (2nd holist assumption)
P6. \( P2 \) contradicts \( P5 \).

C. \( \therefore \) There are no propositions known *a priori*. (P1-P6, proof by contradiction)

Friedman would reject P5: we’re not free to reject *any* of the assumptions that, when conjoined, yield false predictions. In particular, we must always hold onto the *a priori* ones.